A Data-driven Bidding Model for a Cluster of Price-responsive Consumers of Electricity

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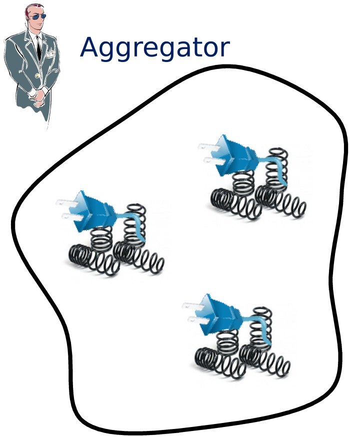
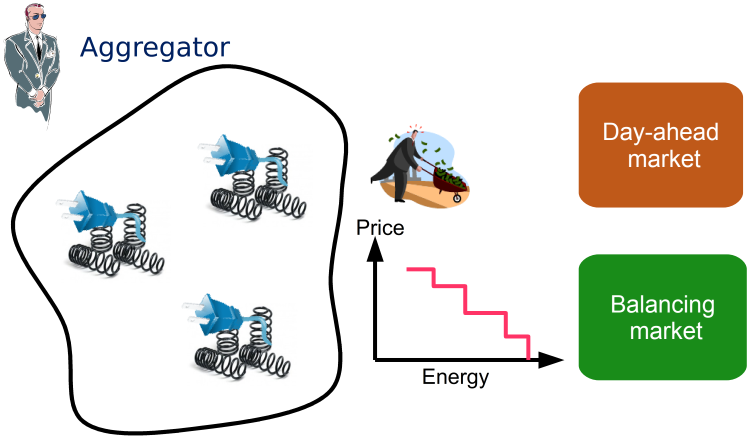
Motivation



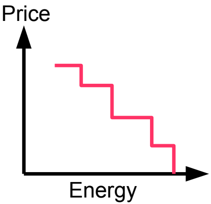
- Price-responsive units (households)

- Too small to participate in the Wholesale electricity market

Motivation


Aggregator
Price
Energy
Day-ahead market
Balancing market

The Bid

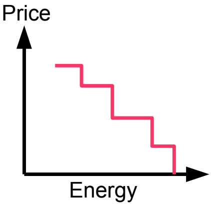


The bid represents the behavior of the aggregated pool in the market.

Parameters θ of the bid:

- Marginal utility ($a_{b,t}$)
- Pick-up and drop-off limits (r_t^u,r_t^d) (equivalent to ramp limits)
- Maximum and minimum power consumption ($\overline{P}_t$,$\underline{P}_t$)

The Bid



Unit-like problem

Maximize (Total utility – cost energy)
x

Pick-up limit (r^u)
Drop-off limit (r^d)
Power bounds ($\underline{P}$,$\overline{P}$)

- The energy assigned to each block is x_{bt}
- And the total estimated load as $x_t^{tot} = \underline{P}_t + \sum_{b \in \mathcal{B}} x_{b,t}$

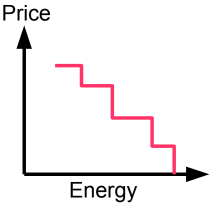
$$\operatorname*{Max}_{x_{b,t}} \sum_{t \in \mathcal{T}} \left(\sum_{b \in \mathcal{B}} a_{b,t} x_{b,t} - \mathrm{price}_t \sum_{b \in \mathcal{B}} \mathrm{x}_{b,t} \right)$$

Subject to

$$-r_t^d \leq x_t^{tot} - x_{t-1}^{tot} \leq r_t^u \qquad t \in \mathcal{T}_{-1}$$

$$0 \leq x_{b,t} \leq \frac{\overline{P}_t - \underline{P}_t}{B} \qquad b \in \mathcal{B}, t \in \mathcal{T}$$

The Bid



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$$\underset{x_{b,t}}{\text{Max}} \sum_{t\in\mathcal{T}} \left(\sum_{b\in\mathcal{B}} a_{b,t} x_{b,t} - \mathrm{price_t} \sum_{b\in\mathcal{B}} \mathrm{x_{b,t}} \right)$$

Subject to

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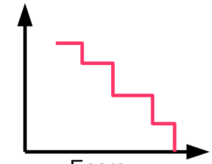
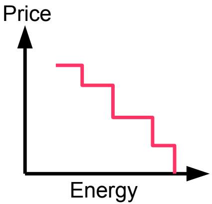
The Bid



Estimate the parameters of the bid using historical data.
The parameters of the bid depend linearly on external factors (i.e., $a_{b,t} = a_b^0 + \sum_{i \in \mathcal{I}} \alpha_i^a Z_{i,t}$)

Time	Price	Load	External Info.
t_1	$price_1$	x_1^{meas}	z_1
t_2	$price_2$	x_2^{meas}	z_2
...	...	...	...

The Bid



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...	...	...	...

Estimation problem: inverse optimization and bilevel programming

Upper-level problem

$$\underset{x,\theta}{\text{Minimize}} \ |x - x^{meas}|$$

$$\theta = \{a_b, r^d, r^u, \underline{P}, \overline{P}\}$$

$$A\theta \leq b$$

Lower-level problem

$$\underset{x}{\text{Maximize}} \ (\text{Total utility} - \text{cost energy})$$

Pick-up limit (r^d)
Drop-off limit (r^u)
Power bounds ($\underline{P}, \overline{P}$)

Upper-level problem

Parameter estimation

$$\underset{x,\theta}{\text{Minimize}} \sum_{t \in \mathcal{T}} w_t \left| x_t^{tot} - x_t^{meas} \right|$$

subject to

$$a_{b,t} \geq a_{b+1,t} \qquad b \in \mathcal{B}, t \in \mathcal{T}$$

KKT conditions of lower-level problem

Upper-level problem

Parameter estimation

$$\underset{x,\theta}{\text{Minimize}} \sum_{t\in\mathcal{T}} w_t \left| x_t^{tot} - x_t^{meas} \right|$$

subject to

$$a_{b,t} \geq a_{b+1,t} \quad b \in \mathcal{B}, t \in \mathcal{T}$$

KKT conditions of lower-level problem

Practical considerations:

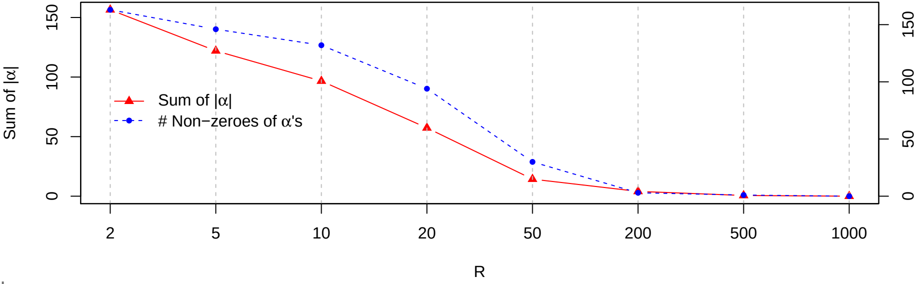
- Remove the absolute value
- w_t: weights, forgetting factor
- Robust constraints to ensure feasibility
- Lasso regularization

LASSO regularization

Add the following term to the objective function

$$R\left(\sum_{i\in\mathcal{I}}\left(|\alpha_i^a| + |\alpha_i^d| + |\alpha_i^{\overline{P}}| + |\alpha_i^{\underline{P}}|\right)\right)$$

- Penalize the affine terms α
- Feature selection & better prediction capabilities
- Choose R by cross validation



Solution Method: Two-step Procedure

Non-linear due to the complementarity constraints

Inverse problem
(relaxed)
Estimate:
$\widehat{\theta} = \{\widehat{a}_{b,t}, \widehat{r}^d_t, \widehat{r}^u_t, \widehat{\overline{P}}_t, \widehat{\underline{P}}_t\}$

$\downarrow$ Fix $\{\widehat{r}^d_t, \widehat{r}^u_t, \widehat{\overline{P}}_t, \widehat{\underline{P}}_t\}$

Refining problem
Re-estimate $a_{b,t}$

Step 1: ***L-penalization*** Solve a linear relaxation of the estimation problem

Step 2: ***Refining problem*** Recompute the parameters defining the utility function with the parameters defining the constraints of the lower-level problem fixed at the values estimated in Step 1

L-Penalization

Penalize violations of the complementarity conditions

$$\begin{aligned}&\underset{x,\lambda}{\text{Minimize}}\ \mathrm{cx}\\&\mathrm{Ax-b}\geq 0\perp\lambda\geq 0\end{aligned}\quad\Longrightarrow\quad\underset{x,\lambda}{\text{Minimize}}\ \mathrm{cx}+\mathrm{L}(\mathrm{Ax-b}+\lambda)$$

- Parameter L penalizes violations of the complementarity constraints
- Optimality is not guaranteed - practical usefulness proved
- Cross-validation to choose L

Solution Method

$$\underset{\substack{x_t,\theta_t,e_t^+,e_t^- \\ \psi_t^{\overline{P}},\psi_t^{\underline{P}},\lambda_t^u,\lambda_t^d \\ \overline{\phi}_{i,t},\underline{\phi}_{i,t},\overline{\varphi}_{i,t},\underline{\varphi}_{i,t},\overline{\eta}_{i,t},\underline{\eta}_{i,t}}}{\text{Minimize}} \quad \overbrace{\sum_{t\in\mathcal{T}} w_t(e_t^+ + e_t^-)}^{\text{Estim error}} + \overbrace{R\left(\sum_{i\in\mathcal{I}}\left(|\alpha_i^u| + |\alpha_i^d| + |\alpha_i^{\overline{P}}| + |\alpha_i^{\underline{P}}|\right)\right)}^{\text{Lasso regularization}} +$$

$$\overbrace{L\left(\sum_{\substack{b\in\mathcal{B}\\ t\in\mathcal{T}}} w_t\left(\psi_{b,t}^{\overline{P}} + \psi_{b,t}^{\underline{P}} + \frac{\overline{P}_t - \underline{P}_t}{B}\right) + \sum_{t\in\mathcal{T}_{-1}} w_t\left(\lambda_t^u + \lambda_t^d + r_t^u + r_t^d\right)\right)}^{\text{Penalization of complementarity constraints}}$$

subject to the following constraints:

1. Upper-level constraints
2. Lower-level constraints
   - Primal feasibility
   - Dual feasibility
   - Stationary conditions

Refining problem

- Reformulate the inverse problem using primal-dual formulation
- In the lower-level, fix the parameters appearing in the constraints
- Substitute the estimated load (x) by the data (x^{meas})

$$\text{Minimize } \ w\epsilon = \text{Weigthed Duality Gap}$$

subject to

$$\text{Primal Ojective} = \text{Dual Objective} + \epsilon$$
$$\text{Primal Constraints}$$
$$\text{Dual Constraints}$$

Refining problem

$$\underset{\substack{a_{b,t},\lambda_t^u,\lambda_t^d,\\ \psi_t^{\overline{P}},\psi_t^{\underline{P}},\underline{\psi}_{b,t},\overline{\psi}_{b,t},\epsilon_t}}{\text{Minimize}} \quad \sum_{t\in\mathcal{T}} w_t \epsilon_t \tag{1}$$

$$\sum_{b\in\mathcal{B}} a_{b,1} x_{b,1}^{meas'} - p_1 \sum_{b\in\mathcal{B}} x_{b,1} + \epsilon_1 = \sum_{b\in\mathcal{B}} \left(\frac{\overline{P}_1 - \underline{P}_1}{B}\right) \overline{\psi}_{b,1} \tag{2a}$$

$$\sum_{b\in\mathcal{B}} a_{b,t} x_{b,t}^{meas'} - p_t \sum_{b\in\mathcal{B}} x_{b,t} + \epsilon_t = \sum_{b\in\mathcal{B}} \left(\frac{\overline{P}_t - \underline{P}_t}{B}\right) \overline{\psi}_{b,t} + \left(r_t^u - \underline{P}_t + \underline{P}_{t-1}\right)\lambda_t^u + \left(r_t^d + \underline{P}_t - \underline{P}_{t-1}\right)\lambda_t^d \qquad t\in\mathcal{T}_{-1} \tag{2b}$$

$$\text{(Stationary conditions lower-level problem)} \tag{2c}$$

$$a_{b,t} \geq a_{b+1,t} \qquad t\in\mathcal{T} \tag{2d}$$

$$\lambda_t^u, \lambda_t^d \geq 0 \qquad t\in\mathcal{T}_{-1} \tag{2e}$$

$$\psi_t^{\overline{P}}, \psi_t^{\underline{P}}, \underline{\psi}_{b,t}, \overline{\psi}_{b,t} \geq 0 \qquad t\in\mathcal{T} \tag{2f}$$

Results

- Data of price-responsive households from Olympic Peninsula project from May 2006 to March 2007.
- The price was sent out every 15 minutes to 27 household
- Decisions made by the home-automation system based on occupancy modes and on price

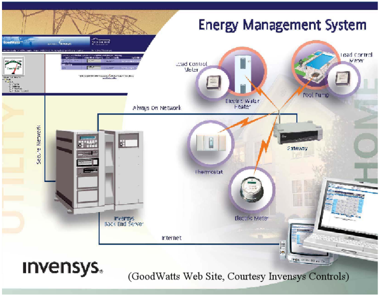
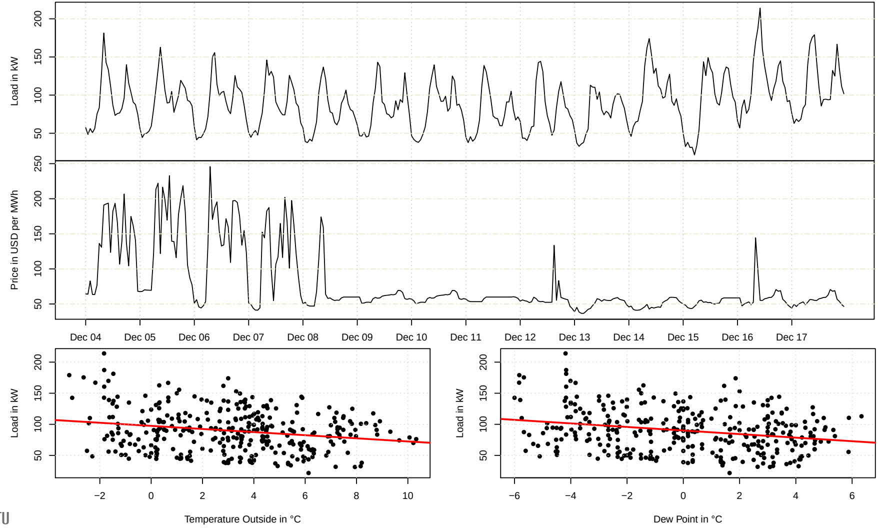


Figure 3.2. Invensys GoodWatts™ System Components

Results

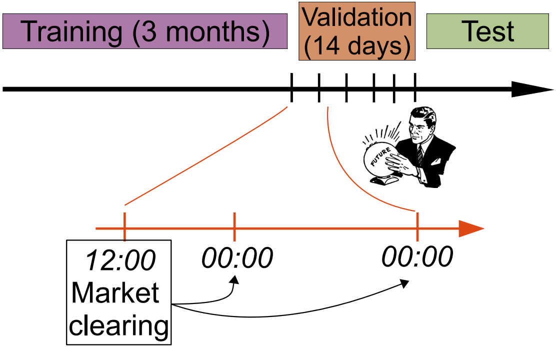
- Load, price, temperature and dew point during december



Results

Cross-validation: In a rolling-horizon manner compute the MAPE

- Penalization parameter L
- Regularization parameter R
- Forgetting factor E



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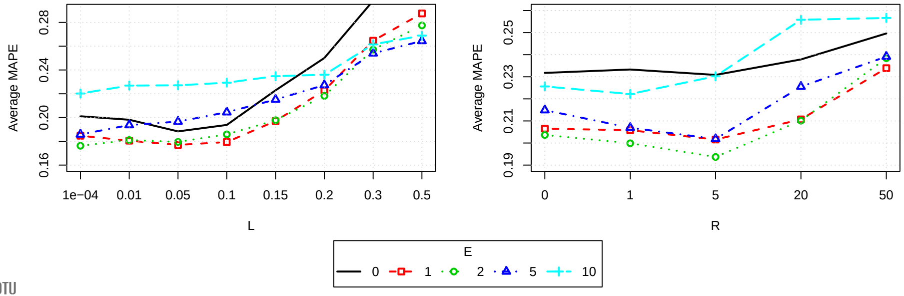
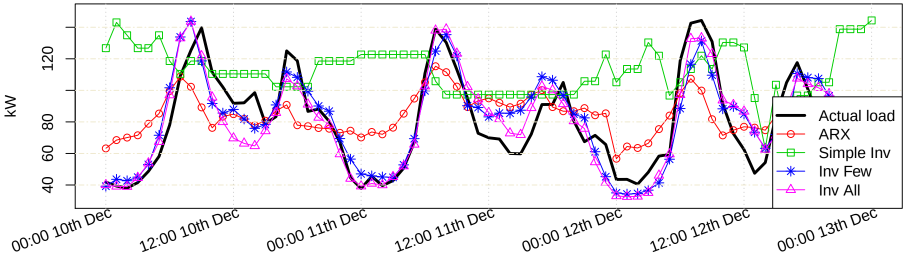


Results

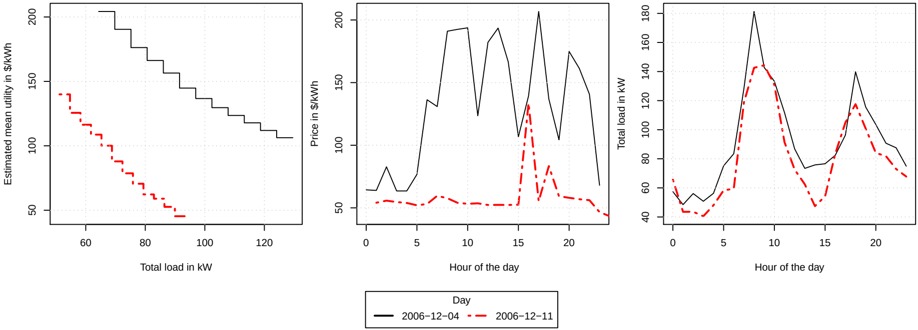
Prediction capabilities of different benchmarked methods



	MAE	RMSE	MAPE
ARX	22.17692	27.50130	0.2752790
Simple Inv	44.43761	54.57645	0.5858138
Inv Few	16.92597	22.27025	0.1846772
Inv All	17.55378	22.39218	0.1987778

Results

Estimated marginal utility for the pool of price-responsive consumers



Summary of the talk

- We capture the **price-response** of the pool of flexible consumers in the form of a **market bid** using price-consumption data.
- We propose a **generalized inverse optimization framework** to estimate the market bid that best captures the price-response of the pool.
- We use machine-learning techniques on a **set of features** to explain the flexibility of the pool
- We test our methodology using data from a **real-world experiment**.

Thank you for listening!

Questions?

A preprint of the associated scientific article can be found in arXiv:
`http://arxiv.org/abs/1506.06587`

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The product of two continuous variables $(\mathrm{Ax} - \mathrm{b})\lambda = 0$ can be reformulated [Siddiqui and Gabriel 2013]:

$$y_1 = 0.5\left((\mathrm{Ax} - \mathrm{b}) + \lambda\right) \tag{3a}$$

$$y_2 = 0.5\left((\mathrm{Ax} - \mathrm{b}) - \lambda\right) \tag{3b}$$

$$y_1^2 - y_2^2 = (\mathrm{Ax} - \mathrm{b})\lambda = 0 \tag{3c}$$

Noting that $\mathrm{Ax} - \mathrm{b} \geq 0$ and $\lambda \geq 0$:

$$y_1 = 0.5\left((\mathrm{Ax} - \mathrm{b}) + \lambda\right) \tag{4a}$$
$$y_2 = 0.5\left((\mathrm{Ax} - \mathrm{b}) - \lambda\right) \tag{4b}$$
$$y_1 = -|y_2| \tag{4c}$$

The absolute value as it is now is not linear. It can be **approximated** by introducing two positive variables y_{2t}^{+} and y_{2t}^{-}

$$y_1 = 0.5\left((\mathrm{Ax} - \mathrm{b}) + \lambda\right) \tag{5a}$$
$$y_2^{+} - y_2^{-} = 0.5\left((\mathrm{Ax} - \mathrm{b}) - \lambda\right) \tag{5b}$$
$$y_1 = -(y_2^{+} + y_2^{-}) \tag{5c}$$
$$y_2^{+}, y_2^{-} \geq 0 \tag{5d}$$

Penalizing $L(y_2^{+} + y_2^{-})$ in the objective function.

$$
\begin{aligned}
y_1 &= 0.5\left((\mathrm{Ax}-\mathrm{b})+\lambda\right) && \text{(6a)}\\
y_2^+ - y_2^- &= 0.5\left((\mathrm{Ax}-\mathrm{b})-\lambda\right) && \text{(6b)}\\
y_1 &= -(y_2^+ + y_2^-) && \text{(6c)}\\
y_2^+, y_2^- &\geq 0 && \text{(6d)}
\end{aligned}
$$

Penalizing $L(y_2^+ + y_2^-)$ in the objective function.

Make a few substitutions and finally obtain that

$$
\begin{aligned}
y_2^+ &= 0.5\left(\mathrm{Ax}-\mathrm{b}\right) && \text{(7a)}\\
y_2^- &= 0.5\left(\lambda\right) && \text{(7b)}
\end{aligned}
$$

Equivalent to penalizing $L(\mathrm{Ax}-\mathrm{b})+\lambda$ in the objective function